Hidden symmetries in quantum mechanics--revealed, by the trajectory-based formulation

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This presentation explores an alternate quantum framework in which the wavefunction $\Psi(t, \mathbf{x})$ plays no role. Instead, quantum states are represented as ensembles of real-valued probabilistic trajectories, $\mathbf{x}(t, \mathbf{C})$, where $\mathbf{C}$ is a trajectory label. Quantum effects arise from the mutual interaction of different trajectories or "worlds," manifesting as partial derivatives with respect to $\mathbf{C}$. The quantum trajectory ensemble $\mathbf{x}(t, \mathbf{C})$ satisfies an action principle, leading to a dynamical partial differential equation (via the Euler-Lagrange procedure), as well as to trajectory-based symmetry and conservation laws (via Noether's theorem). Several of these correspond to standard laws, e.g. conservation of energy. However, one such trajectory-based law (pertaining to curl-free velocity fields) appears to have no standard analog.

A full understanding of the new trajectory-based conservation law may require relativistic considerations. Whereas an earlier, non-relativistic version of the trajectory-based theory turns out to be mathematically equivalent to the time-dependent Schroedinger equation [1–5], the relativistic generalization (for single, spin-zero, massive particles) [6,7,8] is *not* equivalent to the Klein-Gordon (KG) equation—and in fact, avoids certain well-known problems of the latter, such as negative (indefinite) probability density. *It is precisely the new trajectory-based conservation law that makes this possible.* The new relativistic quantum trajectory equations could in principle be used in quantum chemistry calculations, and otherwise could lead to new physical predictions that could be validated or refuted by experiment.

[1] Bouda, A.; From a mechanical Lagrangian to the Schroedinger equation: A modified version of the quantum Newton law, *Int. J. Mod. Phys. A,* **2003**, *18*, 3347–3368.
[2] Holland, P.; Computing the wavefunction from trajectories: particle and wave pictures in quantum mechanics and their relation, *Ann. Phys.*, **2005**, *315*, 505–531.
[3] Poirier, B.; Bohmian mechanics without pilot waves, *Chem. Phys.*, **2010**, *370*, 4–14.
[4] Schiff, J.; Poirier, B.; Communication: Quantum mechanics without wavefunctions, *J. Chem. Phys.*, **2012**, *136*, 031102.
[5] Poirier, B.; The many interacting worlds approach to quantum mechanics, *Phys. Rev. X*, **2014**, *4*, 040002.
[6] Poirier, B.; Trajectory-based theory of relativistic quantum particles, **2012**, *arXiv:1208.6260 [quant-ph]*.
[7] Tsai, H.-M.; Poirier, B.; Exploring the propagation of relativistic quantum wavepackets in the trajectory-based formulation, *J. Phys.*, **2016**, *701*, 012013.
[8] Poirier, B,; Tsai, H.-M.; Trajectory-based Conservation Laws for Massive Spin-zero Relativistic Quantum Particles in 1 + 1 Spacetime, *J. Phys*., **2020**, *1612*, 012022.